

Displacement- and laser-noise-free gravitational-wave detection with two Fabry-Perot cavities

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We propose two Fabry-Perot cavities, each pumped through both the mirrors, positioned in line as a *toy model* of the gravitational-wave (GW) detector free from displacement noise of the test masses. It is demonstrated that the displacement noise of cavity mirrors as well as laser noise can be completely excluded in a proper linear combination of the cavities output signals. We show that in low-frequency approximation (gravitational wave length λ_{gw} is much greater than distance L between mirrors $\lambda_{\text{gw}} \gg L$) the decrease of response signal is about $(L/\lambda_{\text{gw}})^2$, i.e. signal is stronger than the one of the interferometer recently proposed by S. Kawamura and Y. Chen [1].

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I. INTRODUCTION

Currently there is an international “community” of the first generation laser interferometric gravitational wave (GW) detectors [2, 3] (LIGO in USA [4, 5, 6], VIRGO in Italy [7, 8], GEO-600 in Germany [9, 10], TAMA-300 in Japan [11, 12] and ACIGA in Australia [13, 14]). The development of the second-generation GW detectors (Advanced LIGO in USA [15, 16], LCGT in Japan [17]) is underway. The ultimate sensitivity of laser GW detectors is restricted by the Standard Quantum Limit (SQL) — a specific sensitivity level where the measurement noise of the meter (photon shot noise) is equal to its back-action noise (radiation pressure noise) [18, 19, 20, 21]. The sensitivity of GW detectors is also limited by classical displacements noises of various nature: seismic and gravity-gradient noise at low frequencies (below ~ 50 Hz), thermal noise in suspensions, bulk and coatings of the mirrors ($\sim 50 \div 500$ Hz).

In 2004 S. Kawamura and Y. Chen put forward an idea of so called displacement-noise-free interferometer (DFI) which is free from displacement noise of the test masses as well as from optical laser noise [1, 22, 23]. The most attractive feature of DFI is the ability to achieve sub-SQL sensitivity (no SQL since radiation pressure noise is canceled) not accompanied by the necessity to build complicated optical schemes for Quantum-Non-Demolition (QND) measurements [25, 26, 27, 28].

The possibility to separate GW signal from displacement noise of the test masses is based on the *distributed* character of interaction between GW and light wave unlike the localized influence of mirrors positions on the light wave, taking place only at the moments of reflection. The “price” for this separation is the decreased detector response to GWs, especially at low frequencies where the so called long wave approximation is valid,

that is when the distance L separating test masses is much less than the gravitational wave length λ_{gw} , i.e. $L/\lambda_{\text{gw}} \ll 1$ or $\Omega_{\text{gw}}\tau \ll 1$ ($\tau = L/c$ is the light travel time between test masses, c is the speed of light and $\Omega_{\text{gw}} = 2\pi c/\lambda_{\text{gw}}$ is the GW frequency). In particular, the analysis presented in [23] for double Mach-Zehnder interferometer showed that in long wave approximation the shot-noise limited sensitivity to GWs turns out to be limited by $(\Omega_{\text{gw}}\tau)^2$ -factor for 3D configurations and $(\Omega_{\text{gw}}\tau)^3$ -factor for 2D configurations. For signals centered at $\Omega_{\text{gw}}/2\pi \approx 100$ Hz and for interferometers with size of $L \approx 4$ km ($\tau \simeq 10^{-5}$ s), DFI sensitivity of the ground-based detector is $(\Omega_{\text{gw}}\tau)^3 \simeq 10^{-6}$ times worse than that of a conventional single round-trip laser detector.

Another approach to the displacement noise cancellation was presented in [30] where a single detuned Fabry-Perot cavity pumped through both of its movable, partially transparent mirrors was analyzed.

In this paper we investigate model originated from a simple toy model [30] of the GW detector. Our model consists of two double pumped Fabry-Perot cavities positioned in line. Each cavity is pumped through both partially transparent mirrors. By properly combining the signals of output ports of the cavity an experimenter can remove the information about the fluctuations of the mirrors displacements and laser noise from the data. The “price” for isolation of the GW signal from displacement noise in our case is the suppression of sensitivity by factor of $(\Omega_{\text{gw}}\tau)^2$ (resonance gain partially compensates it) as compared with conventional interferometers — it is larger than limiting factor $(\Omega_{\text{gw}}L/c)^3$ of the double Mach-Zehnder 2D configuration [23].

This paper is organized as follows. In Sec. II we analyzed simplified round trip model (without any Fabry-Perot cavities). In Sec. III we derive the response signals of a single double pumped Fabry-Perot cavity to a gravitational wave of arbitrary frequency and introduce their proper linear combination which cancels the laser noise and the fluctuating displacements of one of the mirrors. In Sec. IV we analyze configuration of two

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double-pumped Fabry-Perot cavities which allows to cancel displacement noise of all mirrors completely. Finally in Sec. V we discuss the physical meaning of the obtained results and briefly outline the further prospects.

II. SIMPLIFIED ROUND TRIP MODEL

For clear demonstration we start from analysis of the simplest toy model [1] consisting of 3 platforms A , B and C positioned in line as shown on Fig. 1. GW propagates perpendicularly to this line. We assume that lasers, detectors and mirrors are rigidly mounted on each platform which, in turn, can move as a free masses. We also assume that mean frequency ω_0 of each laser is equal to others. In this section we do not take into account laser noise yet paying attention only on displacement noise and GW signal.

We restrict ourselves to the case when radiation emitted from the laser on some platform is registered (after reflection) by detector on the same platform — so called round trip configuration. Actually detectors are homodyne detectors measuring the phase of incident wave.

Strictly speaking, in order to describe detection of light wave we have to work in the reference frame of detector, i.e. in accelerated frame. However, in our model detector is mounted on the same platform as laser which radiation detector registers and we can work in inertial laboratory frame as it was demonstrated in [30, 31]. Moreover, in this case of round trip configuration we can use transverse-traceless (TT) gauge considering GW action as effective modulation of refractive index $(1+h(t)/2)$ by weak GW perturbation metric $h(t)$. It is worth noting that in the opposite case, when laser and detector are mounted on different platforms, we should use the local Lorentz (LL) gauge — see details in [31].

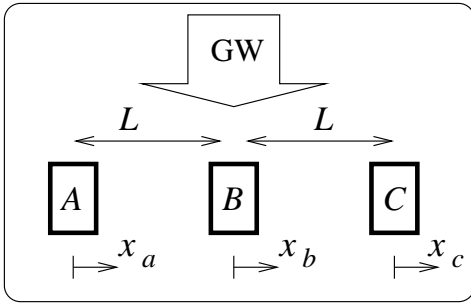


FIG. 1: Simplified model of displacement noise-free detector. On each platforms we place laser, detectors and reflecting mirrors. Mean distances between neighboring platforms are equal to L . GW propagates perpendicularly to line consisting of three platform.

We denote the phase of the wave emitted, for example, from platform A , reflected on platform C and detected on platform A as ϕ_{aca} and so on. Let us measure phase ϕ_{aba} (of the wave emitted from and detected on platform

A after reflection from platform B) and phase ϕ_{bab} (see also Fig. 1)

$$\phi_{aba}(t) = \psi_h(t) + k[2x_b(t-\tau) - x_a(t) - x_a(t-2\tau)], \quad (1)$$

$$\phi_{bab}(t) = \psi_h(t) + k[-2x_a(t-\tau) + x_b(t) + x_b(t-2\tau)], \quad (2)$$

$$\psi_h(t) \equiv \frac{\omega_0}{2} \int_{t-2\tau}^t h(t') dt', \quad (3)$$

Here $k = \omega_0/c$ is the wave vector of light emitted by laser, $\tau = L/c$ is bouncing time and $h(t)$ is perturbation of dimensionless metric originated by GW, c is the speed of light.

Obviously, we can exclude information on displacement x_a of platform A in the following combination \tilde{C}_1 :

$$\begin{aligned} \tilde{C}_1(t) &= 2\phi_{aba} - \phi_{bab}(t+\tau) - \phi_{bab}(t-\tau) = \\ &= 2\psi_h(t) - \psi_h(t+\tau) - \psi_h(t-\tau) + \\ &\quad + k[2x_b(t-\tau) - x_b(t+\tau) - x_b(t-3\tau)]. \end{aligned} \quad (4)$$

Exclusion of information on displacements of platforms A in combination \tilde{C}_1 means that we effectively convert platform A into ideal (i.e. displacement noise free) test mass for GW detection.

By similar way measuring phases ϕ_{bcb} and ϕ_{cbc}

$$\begin{aligned} \phi_{cbc}(t) &= \psi_h(t) + k[-2x_b(t-\tau) + x_c(t) + x_c(t-2\tau)], \\ \phi_{bcb}(t) &= \psi_h(t) + k[2x_c(t-\tau) - x_b(t) - x_b(t-2\tau)], \end{aligned}$$

we can exclude information on displacement x_c of platform C in combination \tilde{C}_2 :

$$\begin{aligned} \tilde{C}_2(t) &= 2\phi_{cbc}(t) - \phi_{bcb}(t-\tau) - \phi_{bcb}(t+\tau) = \\ &= 2\psi_h(t) - \psi_h(t-\tau) - \psi_h(t+\tau) + \\ &\quad + k[-2x_b(t-\tau) + x_b(t+\tau) + x_b(t-3\tau)] \end{aligned} \quad (5)$$

Comparing (4) and (5) we see that position x_b makes contributions into \tilde{C}_1 and \tilde{C}_2 with opposite signs — in contrast to the GW signal. So we should just sum \tilde{C}_1 and \tilde{C}_2 in order to exclude *completely* information on positions of all platforms:

$$\tilde{C}_3(t) = \frac{\tilde{C}_1(t) + \tilde{C}_2(t)}{2} = 2\psi_h(t) - \psi_h(t+\tau) - \psi_h(t-\tau) \quad (6)$$

It is useful to rewrite this formula in frequency domain:

$$\psi_h(\Omega) = \omega_0 \tau h(\Omega) e^{i\Omega\tau} \frac{\sin \Omega\tau}{\Omega\tau} \quad (7)$$

$$\tilde{C}_3(\Omega) = -\omega_0 \tau h(\Omega) (1 - e^{i\Omega\tau})^2 \left(\frac{\sin \Omega\tau}{\Omega\tau} \right) \quad (8)$$

In long wave approximation ($\Omega\tau \ll 1$) we have in time and frequency domain correspondingly

$$\tilde{C}_3(t) \simeq -\omega_0 \tau^3 \ddot{h}(t), \quad (9)$$

$$\tilde{C}_3(\Omega) \simeq \omega_0 \tau (\Omega \tau)^2 h(\Omega) \quad (10)$$

We see that in our simplest model the payment for separation of GW signal from displacement noise is decrease of GW response, which in long wave approximation is about $(\Omega \tau)^2$.

III. RESPONSE OF DOUBLE PUMPED FABRY-PEROT CAVITY TO A GRAVITATIONAL WAVE

Now we can analyze model with two Fabry-Perot cavities. We start from single double pumped Fabry-Perot cavity presented on Fig. 2. Pump waves in different input ports are assumed to be orthogonally polarized in order the corresponding output waves to be separately detectable and to exclude nonlinear coupling of the corresponding intracavity waves. To simplify our model we assume that mirrors and lasers with detectors of each cavity are rigidly mounted on two movable platform (see Fig. 2) (in contrast to scheme analyzed in [30] with four platforms). Laser L_1 with its detectors and mirror with amplitude transmittance T_1 are rigidly mounted on movable platform P_1 . In other words, we assume that all the elements on the platform do not move with respect to each other. Laser L_1 pumps the cavity from the left and we assume that the wave transmitted through the cavity is redirected to platform P_1 by reflecting mirror R_2 as shown on Fig 2a. So waves, emitted by this laser, are finally registered by detectors positioned on the same platform as laser. The mirror with amplitude transmittance T_2 and laser L_2 pumping cavity from the right with its detectors are rigidly mounted on platform P_2 . We assume that amplitude transmission coefficients of mirrors are small: $T_1, T_2 \ll 1$. We put mean distance between the mirrors to be equal to L . Without the loss of generality we assume the cavity to be lying in the plane perpendicular to direction of GW and along one of the GW principal axes.

It is convenient to represent the electric field operator of the light wave as a sum of (i) the “strong” (classical) plane monochromatic wave (which approximates the light beam with cross-section S) with amplitude A and frequency ω_0 and (ii) the “weak” wave describing quantum fluctuations of the electromagnetic field:

$$E(x, t) = \sqrt{\frac{2\pi\hbar\omega_0}{Sc}} \left[A + a(x, t) \right] e^{-i(\omega_0 t \mp k_0 x)} + \text{h.c.}, \quad (11a)$$

$$a(x, t) = \int_{-\infty}^{+\infty} a(\omega_0 + \Omega) e^{-i\Omega(t \mp x/c)} \frac{d\Omega}{2\pi},$$

with amplitude $a(\omega_0 + \Omega)$ (Heisenberg operator to be strict) obeying the commutation relations:

$$\begin{aligned} [a(\omega_0 + \Omega), a(\omega_0 + \Omega')] &= 0, \\ [a(\omega_0 + \Omega), a^\dagger(\omega_0 + \Omega')] &= 2\pi\delta(\Omega - \Omega'). \end{aligned}$$

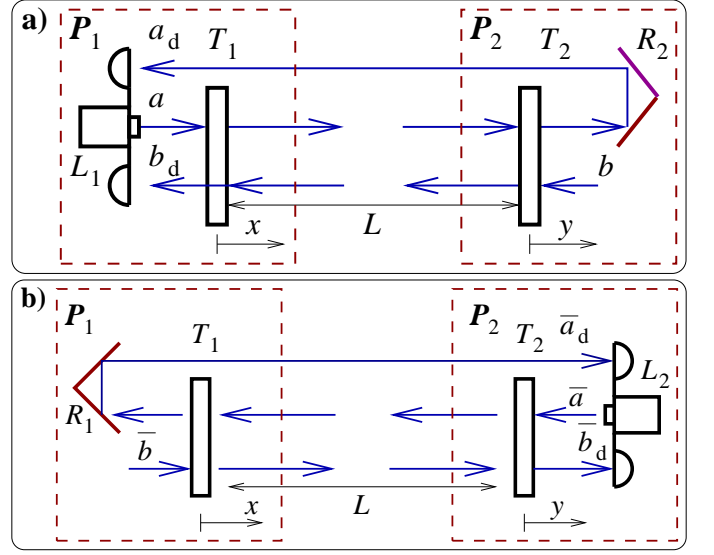


FIG. 2: Emission-detection scheme of *one* double pumped Fabry-Perot cavity. a) pump by laser L_1 through the left port is shown only. Pump laser with both detectors and input mirror are assumed to be rigidly mounted on moveable platform P_1 . Transmitted wave is redirected by additional mirror R_2 to platform P_1 . Transmitted and reflected wave are detected by detectors on platform P_1 . End and additional mirror R_2 are assumed to be rigidly mounted on movable platform P_2 . b) pump by laser L_2 through the right port of the same cavity with its detectors and redirecting mirror R_1 is shown.

For briefness throughout the paper we denote

$$a \equiv a(\omega_0 + \Omega), \quad a_-^\dagger \equiv a^\dagger(\omega_0 - \Omega)$$

This notation for quantum fluctuations a is convenient since it coincides exactly with the Fourier representation of the classical fields. and we omit the $\sqrt{2\pi\hbar\omega_0/Sc}$ multiplier. For convenience throughout the paper we denote mean amplitudes by block letters and corresponding small additions by *the same* small letter as in (11). In ideal case the input laser wave is in coherent state (it means that fluctuational amplitude $a(\omega_0 + \Omega)$ describes vacuum fluctuations). In more realistic case small amplitudes a, a^\dagger describes technical laser fluctuations. But fluctuational wave incoming into cavity through the non-pumped port (denoted by b or \bar{b} on Fig. 2) is always in vacuum state.

In our model, as in simplified model analyzed in previous section, detectors are mounted on the same platform as laser which radiation detectors register and we can work in inertial laboratory frame [30, 31] considering GW action as effective modulation of refractive index $(1 + h(\Omega)/2)$ by weak GW perturbation metric $h(\Omega)$.

First, we consider pump by laser L_1 shown in the Fig. 2a. Using calculations presented in Appendix A we can write down formulas for small complex amplitudes a_d, b_d of waves detected on platform P_1 (see notations

on Fig. 2a):

$$a_d = -\theta_0 \psi [T a + \mathcal{R}_2 b] + \frac{i T_2 \vartheta_0^2}{1 - R_1 R_2 \vartheta_0^2 \psi^2} \left(\frac{1 + \psi^2}{2} u_x - \psi(u_y + u_h) \right), \quad (12)$$

$$b_d = \mathcal{R}_1 a + T b + \frac{i T_1 R_2 \vartheta_0^2}{1 - R_1 R_2 \vartheta_0^2 \psi^2} \left(\frac{1 + \psi^2}{2} u_x - \psi(u_y + u_h) \right), \quad (13)$$

$$\text{where } \psi = e^{i\Omega\tau}, \quad \theta_0 = e^{i\delta\tau}, \quad \tau = \frac{L}{c}. \quad (14)$$

Here fluctuational amplitudes a and b describe laser noise and vacuum fluctuations correspondingly, δ is detuning between laser frequency and resonance frequency of cavity. $R_1 = \sqrt{1 - T_1^2}$, $R_2 = \sqrt{1 - T_2^2}$ are reflectivities of mirrors, by calligraph letters we denote coefficients of cavity's transparency and reflectivities:

$$\mathcal{T} = \frac{-\vartheta_0 \psi T_1 T_2}{1 - R_1 R_2 \vartheta_0^2 \psi^2}, \quad (15)$$

$$\mathcal{R}_1 = \frac{R_2 \vartheta_0^2 \psi^2 - R_1}{1 - R_1 R_2 \vartheta_0^2 \psi^2}, \quad \mathcal{R}_2 = \frac{R_1 \vartheta_0^2 \psi^2 - R_2}{1 - R_1 R_2 \vartheta_0^2 \psi^2} \quad (16)$$

The influence of fluctuational (non-geodesic) displacements x , y in (12, 13) (to be strict its Fourier representations) is described by values u_x , u_y :

$$u_x = A_{in} 2ikx(\Omega), \quad u_y = A_{in} 2iky(\Omega), \quad (17)$$

$$A_{in} = \frac{i T_1 A}{1 - R_1 R_2 \vartheta_0^2}, \quad k = \frac{\omega_0}{c}, \quad (18)$$

where A_{in} is mean amplitude of wave circulating inside the cavity, we assume A_{in} to be real (see also Fig. 4 in Appendix A), A is mean amplitude of wave emitted by laser L_1 (to be strict amplitude of wave falling on mirror with transparency T_1). Interaction of light with GW in (12, 13) is described by dimensionless metric perturbation h through value u_h :

$$u_h = A_{in} ikL h(\Omega) \frac{\sin \Omega\tau}{\Omega\tau}. \quad (19)$$

It is worth emphasizing that both output waves a_d , b_d contain the identical information on displacements and GW signal — see formulas (12, 13). However, terms describing laser fluctuations have different coefficients at laser noise amplitude a . Hence, we can take such linear combination of two detectors output signals which does not contain laser noise (but it will contain the information on GW signal and displacements). Recall, that in fact we have the homodyne detectors, which can measure arbitrary quadrature component of output waves (with pump laser used as a local oscillator). Our analysis shows that complete cancellation of laser noise is possible at two conditions: i) we should measure the same quadrature in both detector ports; ii) detuning should be zero. For zero detuning only phase quadrature contains information on GW signal and displacements (amplitude

quadratures are free from GW signal in linear approximation). Therefore, below we consider the case of detecting phase quadratures at zero detuning. For phase quadrature one can obtain the following formulas (see details in Appendix A)

$$a_d^{(p)} = \frac{T_1 T_2 \psi^2 (a + a_-^\dagger) - \psi(R_1 \psi^2 - R_2)(b + b_-^\dagger)}{\sqrt{2}(1 - R_1 R_2 \psi^2)} + \frac{2iT_2}{\sqrt{2}(1 - R_1 R_2 \psi^2)} \left(\frac{1 + \psi^2}{2} u_x - \psi(u_y + u_h) \right); \quad (20)$$

$$b_d^{(p)} = \frac{(R_2 \psi^2 - R_1)(a + a_-^\dagger) - T_1 T_2 \psi(b + b_-^\dagger)}{\sqrt{2}(1 - R_1 R_2 \psi^2)} + \frac{2iT_1 R_2}{\sqrt{2}(1 - R_1 R_2 \psi^2)} \left(\frac{1 + \psi^2}{2} u_x - \psi(u_y + u_h) \right). \quad (21)$$

We see that laser noise amplitudes contribute to output amplitude quadratures $a_d^{(a)}$, $b_d^{(a)}$ in the same combination $(a + a_-^\dagger)$. Hence, we take linear combinations $C_{ph} = S_a(\Omega)a_d^{(a)} + S_b(\Omega)b_d^{(a)}$ and in order to exclude technical laser noise we specify weight coefficients S_a , S_b as following:

$$S_a(\Omega) = \frac{R_2 \psi^2 - R_1}{1 - R_1 R_2 \psi^2}, \quad S_b(\Omega) = \frac{-T_1 T_2 \psi^2}{1 - R_1 R_2 \psi^2}, \quad (22)$$

$$C_{ph} = \frac{(\psi^2 - R_1 R_2)\psi(b + b_-^\dagger)}{\sqrt{2}(1 - R_1 R_2 \psi^2)} + \frac{-2iT_2 R_1}{\sqrt{2}(1 - R_1 R_2 \psi^2)} \left(\frac{1 + \psi^2}{2} u_x - \psi(u_y + u_h) \right). \quad (23)$$

Here we use normalization $|S_a|^2 + |S_b|^2 = 1$. So we completely cancel laser noise (i.e. combination C_{ph} contains no term proportional to $(a + a_-^\dagger)$, only vacuum noise $\sim (b + b_-^\dagger)$ present).

The dependence of weight coefficients S_a , S_b on frequency Ω mean that before summation output currents of homodyne detectors registering phase quadratures $a_d^{(p)}$, $b_d^{(p)}$ should be passed through filters with transmission coefficients $S_a(\Omega)$, $S_b(\Omega)$ correspondingly.

Now we can write down formulas for output fields pumping by laser L_2 from opposite port (see Fig. 2b). We assume that radiation from laser L_2 is polarized normally to radiation emitted by laser L_1 . We denote all values by the same letters as above but mark them by bar. For simplicity we assume that excited by laser L_2 mean amplitude \bar{A}_{in} of the wave circulating inside the cavity is equal to A_{in} : $\bar{A}_{in} = A_{in}$. Also we assume that laser L_2 is also tuned in resonance (i.e. $\bar{\delta} = 0$) and we measure phase quadratures in corresponding output waves. Again we take corresponding combination \bar{C}_{ph} to exclude laser noise. Then by using the following substitutions:

$$T_{1,2} \rightarrow T_{2,1}, \quad R_{1,2} \rightarrow R_{2,1}, \\ u_x \rightarrow -u_y, \quad u_y \rightarrow -u_x, \quad u_h \rightarrow u_h$$

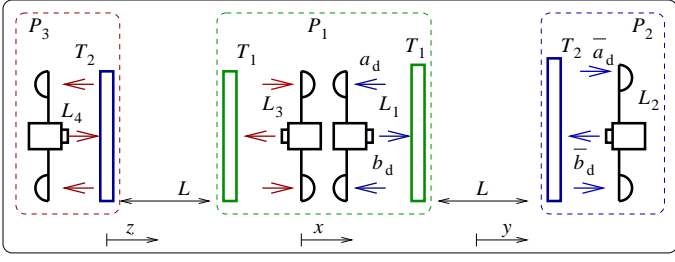


FIG. 3: Configuration of two doubled pumped Fabry-Perot cavities. The right Fabry-Perot cavity is the same as shown on Fig. 2, the redirecting mirrors are not shown. The left Fabry-Perot cavity is identical to right cavity having the mirror with transparency T_1 rigidly mounted on platform P_1 . Left cavity is pumped by lasers L_3 and L_4 , redirecting mirrors are also not shown.

we rewrite formula (23) for combination \bar{C}_{ph}

$$\bar{C}_{ph} = \frac{(\psi^2 - R_1 R_2) \psi (\bar{b} + \bar{b}^\dagger)}{\sqrt{2}(1 - R_1 R_2 \psi^2)} + \frac{-2iT_1 R_2}{\sqrt{2}(1 - R_1 R_2 \psi^2)} \left(\frac{1 + \psi^2}{2} (-u_y) + \psi(u_x + u_h) \right). \quad (24)$$

IV. DISPLACEMENTS EXCLUSION IN CONFIGURATION OF TWO DOUBLE PUMPED FABRY-PEROT CAVITIES

Comparing formulas (23) and (24) we see that platform displacements (u_x and u_y) make different contributions. It allows to exclude, for example, displacement y (u_y) in the following combination:

$$C_1 = \sqrt{\frac{T_1 R_2}{T_2 R_1}} \frac{1 + \psi^2}{2} C_{ph} - \sqrt{\frac{T_2 R_1}{T_1 R_2}} \psi \bar{C}_{ph} = g_{1 \text{ vac}} - \frac{\sqrt{2}i\sqrt{T_1 T_2 R_1 R_2}}{(1 - R_1 R_2 \psi^2)} \left(\left[\frac{1 - \psi^2}{2} \right]^2 u_x - \frac{\psi(1 - \psi)^2}{2} u_h \right), \quad (25)$$

$$g_{1 \text{ vac}} \equiv \frac{(\psi^2 - R_1 R_2) \psi}{1 - R_1 R_2 \psi^2} \left(\sqrt{\frac{T_1 R_2}{T_2 R_1}} \frac{1 + \psi^2}{2} \frac{(\bar{b} + \bar{b}^\dagger)}{\sqrt{2}} - \sqrt{\frac{T_2 R_1}{T_1 R_2}} \psi \frac{(\bar{b} + \bar{b}^\dagger)}{\sqrt{2}} \right). \quad (26)$$

Here we denote by $g_{1 \text{ vac}}$ the linear combination of vacuum fluctuations b and \bar{b} incoming into cavity through non-pumped ports.

It is a very important result — exclusion of information on u_y is equivalent to conversion of platform P_2 into ideal mass, which is free from fluctuational displacement y . The price for such conversion is decrease of GW response by factor approximately $\sim (1 - \psi)^2$ (it is about $\sim (\Omega\tau)^2$ in long wave approximation) as compared with conventional laser GW detector.

Now we have to exclude information on u_x (i.e. displacement x of platform P_1). It can be done in configuration of two double pumped Fabry-Perot cavities. Let us add second Fabry-Perot cavity (left cavity on Fig. 3) positioned in line with first cavity considered above. For simplicity we assume that parameters of both cavities are identical and that amplitudes and detunings of lasers L_3, L_4 pumped second cavity are the same as of lasers L_1, L_2 correspondingly. Due to place shortage on Fig. 3 we could not show redirected mirrors assuming that complete scheme for each Fabry-Perot cavity is the same as shown on Fig. 2 for one cavity. The mirrors with transparency T_1 and lasers L_1, L_3 with its detectors are rigidly mounted on the same platform P_1 . The other mirror of second cavity and laser L_4 with its detectors are rigidly mounted on platform P_3 , we denote its position by z . We also assume that lasers L_3 and L_4 are tuned in resonance with second cavity and we measure phase quadrature components by corresponding homodyne detectors.

In order to calculate formulas for phase quadratures of output waves $e_d^{(p)}, \bar{e}_d^{(p)}$ of second cavity just rewriting formulas (20, 21) for phase quadratures $a_d^{(p)}, \bar{a}_d^{(p)}$ we apply following substitutions:

$$u_y \rightarrow -u_z, \quad u_x \rightarrow -u_x, \quad u_h \rightarrow u_h, \quad (27a)$$

$$b \rightarrow e, \quad \bar{b} \rightarrow \bar{e}. \quad (27b)$$

Here amplitudes e, \bar{e} describe corresponding vacuum noise incoming into second Fabry-Perot cavity through non-pumped ports.

The noise from lasers L_3, L_4 we exclude by the same manner as for first cavity. We can also exclude information on displacement z in combination C_2 by the same way as we excluded displacement y in combination C_1 . One can write this combination C_2 free from displacement z using substitutions (27):

$$C_2 = g_{2 \text{ vac}} - \frac{\sqrt{2}i\sqrt{T_1 T_2 R_1 R_2}}{(1 - R_1 R_2 \psi^2)} \left(- \left[\frac{1 - \psi^2}{2} \right]^2 u_x - \frac{\psi(1 - \psi)^2}{2} u_h \right). \quad (28)$$

Here $g_{2 \text{ vac}}$ is the combinations of vacuum noise amplitudes e, \bar{e} , described by the same formula (26) with only substitutions $b \rightarrow e, \bar{b} \rightarrow \bar{e}$.

Comparing (25, 28) we see that value u_x makes contributions into C_1 and C_2 with the opposite signs, whereas GW contributions (i.e. u_h) have the same sign (it is obvious consequence of tidal nature of GW). So in order to exclude u_x we should just sum C_1 and C_2 :

$$C_{\text{DFI}} = \frac{C_1 + C_2}{\sqrt{2}} = \frac{g_{1 \text{ vac}} + g_{2 \text{ vac}}}{\sqrt{2}} + \frac{i\sqrt{T_1 T_2 R_1 R_2}}{(1 - R_1 R_2 \psi^2)} \left(\psi(1 - \psi)^2 u_h \right). \quad (29)$$

Comparing combination C_{DFI} with combination C_{ph} (23) we see that gravitational signal in C_{DFI} is smaller by factor $(1 - \psi)^2$ which in approximation of long gravitational

wave length $L/\lambda_{\text{grav}} \ll 1$ (or $\Omega\tau \ll 1$) is about $(\Omega\tau)^2$. It is the same decrease of GW response as in combination \tilde{C}_3 (8, 10)) for simplified model considered in Sec. II (the only difference is the presence of resonance gain in (29)).

Assuming $T_1, T_2 \ll 1$ and $\Omega\tau \ll 1$ we rewrite C_{DFI} in narrow band approximation:

$$C_{\text{DFI}} \simeq \frac{g_{1\text{vac}} + g_{2\text{vac}}}{\sqrt{2}} + \frac{\sqrt{T_1^3 T_2}}{T_1^2 + T_2^2} \frac{\Omega^2 \tau}{\gamma - i\Omega} A k L h(\Omega), \quad (30)$$

where $\gamma = (T_1^2 + T_2^2)/4\tau$ is the relaxation rate (half bandwidth) of Fabry-Perot cavity.

Recall that in a simplest detector with two test masses and only one round trip of light between them gravitational signal is about $AkLh$ with the same value of fluctuational field. So assuming in (30) that $\gamma \approx \Omega$ and $T_1 \approx T_2$ we see that signal-to-noise ratio of our cavities operating as displacement noise free detector is smaller by factor about $\sim \Omega\tau$ as compared with simplest detector.

V. CONCLUSION

In this paper we have analyzed the operation of two Fabry-Perot cavities positioned in line, performing the displacement-noise-free gravitational-wave detection. We have demonstrated that it is possible to construct a linear combination of four response signals which cancels *displacement fluctuations* of the mirrors. At low frequencies the GW response of our cavities turns out to be better than that of the Mach-Zehnder-based DFIs [23] due to the different mechanisms of noise cancellation.

Due to reflected and transmitted waves carry the same information on mirrors displacement we have additional possibility to exclude *laser noise* (of course, fundamental vacuum noise can not be not excluded).

We show that considered DFI with two Fabry-Perot cavities is similar to the simplest round trip configuration shown in Fig. 1.

For simplicity we have analyzed three platform configuration. The configurations with larger number of movable platform is more realistic and it may provide better sensitivity. For example, the middle platform may be splitted into three platforms: two platforms with mirrors (having transparency T_1) and one platform between them (with lasers L_1, L_3 and its detectors). Variants of such configurations are under investigation now.

The proposed configuration of DFI may be a promising candidate for the future generation of GW detectors with displacement and laser noise exclusion which, in turn, will allow to overcome standard quantum limit.

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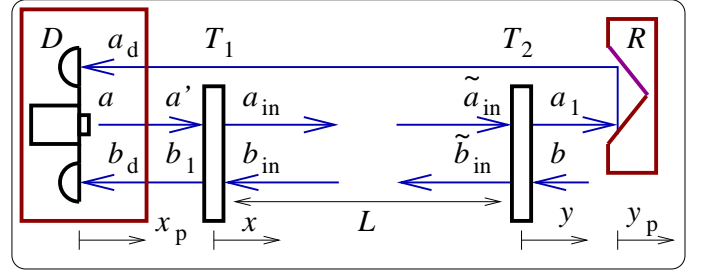


FIG. 4: Detailed scheme of measurement (generalization of shown on the Fig. 2a). Cavity mirrors are movable, laser and detectors are placed on detecting platform D, additional mirror is placed on reflecting platform R.

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APPENDIX A: DERIVATION OF FORMULAS FOR FABRY PEROT CAVITY

In this Appendix we derive formulas (12, 13) for complex amplitudes and (20, 21) for phase quadratures for single Fabry-Perot cavity pumped by laser from the left.

For methodical purpose we start from general case when laser with detectors, mirrors and additional mirror are mounted on separated rigid movable platform each as shown on Fig. 4. Below we use notations on Fig. 4. First we find complex mean amplitudes, writing boundary conditions on right and left mirror:

$$\tilde{B}_{in} = -R_2 \tilde{A}_{in}, \quad A_1 = i \tilde{A}_{in} T_2, \\ A_{in} = i T_1 A - R_1 B_{in}, \quad B_1 = i T_1 B_{in} - R_1 A$$

From these equations and obvious relations $\tilde{A}_{in} = A_{in} \vartheta_0$ and $B_{in} = \tilde{B}_{in} \vartheta_0$ one can find formula (18) for A_{in} and for mean output fields:

$$B_1 = \frac{A(R_2 \vartheta_0^2 - R_1)}{1 - R_1 R_2 \vartheta_0^2}, \quad A_1 = \frac{-T_1 T_2 \vartheta_0 A}{1 - R_1 R_2 \vartheta_0^2}. \quad (A1)$$

To find small amplitudes inside cavity we write down boundary condition on right and left mirrors correspondingly:

$$\tilde{b}_{in} = -R_2 \tilde{a}_{in} + i T_2 b - R_2 \vartheta_0 u_y \quad (A2)$$

$$a_{in} = i T_1 a' - R_1 b_{in} - R_1 R_2 \vartheta_0^2 u_x. \quad (A3)$$

And taking into account GW action as effective variation of refractive index $1 + h/2$

$$\tilde{a}_{in} = \vartheta_0 \psi a_{in} + A_{in} \vartheta_0 i j(\Omega), \quad (A4)$$

$$b_{in} = \vartheta_0 \psi \tilde{b}_{in} - R_2 A_{in} \vartheta_0^2 i j(\Omega), \quad (A5)$$

$$j = \frac{\omega_0 h(\Omega)}{2} \int_{t-\tau}^t e^{-i\Omega(t'-t)} dt' = \frac{k L h}{2} \left(\frac{1 - e^{i\Omega\tau}}{-i\Omega\tau} \right),$$

$$\psi u_h = A_{in}(1 + \psi)j$$

we find small amplitudes inside cavity:

$$a_{in} = \frac{iT_1 a' - iT_2 R_1 \vartheta_0 \psi b}{1 - R_1 R_2 \vartheta_0^2 \psi^2} + \frac{R_1 R_2 \vartheta_0^2 (\psi(u_y + u_h) - u_x)}{1 - R_1 R_2 \vartheta_0^2 \psi^2}, \quad (A6)$$

$$b_{in} = \frac{-iT_1 R_2 \vartheta_0^2 \psi^2 a' + iT_2 \vartheta_0 \psi b}{1 - R_1 R_2 \vartheta_0^2 \psi^2} + \frac{R_2 \vartheta_0^2 \psi (R_1 R_2 \vartheta_0^2 \psi u_x - u_y - u_h)}{1 - R_1 R_2 \vartheta_0^2 \psi^2}. \quad (A7)$$

Now using second boundary condition on right mirror we can find transmitted wave a_1 :

$$\begin{aligned} a_1 &= -R_2 b + iT_2 \tilde{a}_{in} = \\ &= R_2 b + T a' - \frac{iR_1 R_2 T_2 \vartheta_0^3 \psi}{1 - R_1 R_2 \vartheta_0^2 \psi^2} (u_x - \psi(u_y + u_h)). \end{aligned} \quad (A8)$$

By the same manner from second boundary condition on left mirror we find reflected wave b_1

$$\begin{aligned} b_1 &= -R_1 a' + iT_1 b_{in} + (-R_1 A) 2ikx = \\ &= R_1 a' + T b + \frac{i(R_1 - R_2 \vartheta_0^2)}{T_1} u_x + \\ &\quad + \frac{iT_1 R_2 \vartheta_0^2}{1 - R_1 R_2 \vartheta_0^2 \psi^2} (u_x - \psi(u_y + u_h)). \end{aligned} \quad (A9)$$

Here we write formula for b_1 in this form in order to extract term proportional to same combinations of mirrors positions as in (A8).

In order to express fields a_1 , b_1 through small amplitude a describing laser fluctuations we should substitute in (A8, A9)

$$a' = (a - A i k x_p). \quad (A10)$$

Now we can find field b_d falling on detector

$$b_d = b_1 - B_1 i k x_p = R_1 a + T b + \frac{i(R_1 - R_2 \vartheta_0^2)}{T_1} u_x +$$

$$\begin{aligned} &+ \frac{iT_1 R_2 \vartheta_0}{1 - R_1 R_2 \vartheta_0^2} (\vartheta_0 u_x - \vartheta(u_y + u_h)) - \\ &- \frac{A_{in}(1 - R_1 R_2 \vartheta_0^2) i k x_p}{iT_1} \left(\frac{R_2 \vartheta_0^2 - R_1}{1 - R_1 R_2 \vartheta_0^2} + \frac{R_2 \vartheta_0^2 - R_1}{1 - R_1 R_2 \vartheta_0^2} \right). \end{aligned} \quad (A11)$$

Using (A8) for transmitted wave a_1 we find formula for amplitude a_d falling on detector:

$$\begin{aligned} a_d &= \vartheta(-a_1 - A_1 2iky_p) + (-\vartheta_0 A_1) i j - (-\vartheta_0 A_1) i k x_p = \\ &= -\vartheta_0 \psi (T a + R_2 b) + \frac{iT_2 R_1 R_2 \vartheta_0^4 \psi^2 (u_x - \psi(u_y + u_h))}{1 - R_1 R_2 \vartheta_0^2 \psi^2} - \\ &\quad - iT_2 \vartheta_0^2 \psi (u_h + A_{in} 2iky_p) + \\ &\quad + iT_2 \vartheta_0^2 A_{in} i k x_p \left(1 + \psi^2 \frac{(1 - R_1 R_2 \vartheta_0^2)}{(1 - R_1 R_2 \vartheta_0^2 \psi^2)} \right). \end{aligned} \quad (A12)$$

Now substituting $x_p = x$ and $y_p = y$ into (A11, A12) one can obtain formulas (12, 13).

It is useful to rewrite formulas (A11, A12) for particular case of zero detuning ($\vartheta_0 = 1$) and $x_p = x$ and $y_p = y$:

$$a_d = \frac{T_1 T_2 \psi^2 a - \psi(R_1 \psi^2 - R_2) b_2}{1 - R_1 R_2 \psi^2} + \quad (A13)$$

$$\begin{aligned} &+ \frac{iT_2}{1 - R_1 R_2 \psi^2} \left(\frac{1 + \psi^2}{2} u_x - \psi(u_y + u_h) \right), \\ b_d &= \frac{(R_2 \psi^2 - R_1) a - T_1 T_2 \psi b_2}{1 - R_1 R_2 \psi^2} + \\ &\quad + \frac{iT_1 R_2}{1 - R_1 R_2 \psi^2} \left(\frac{1 + \psi^2}{2} u_x - \psi(u_y + u_h) \right). \end{aligned} \quad (A14)$$

We define phase quadratures of fields falling on detectors as following:

$$\begin{aligned} a_d^{(p)} &\equiv \frac{-A_d^* a_d + A_d a_{d-}^\dagger}{i|A_d|\sqrt{2}} = \frac{(\vartheta_0^*)^2 a_d + \vartheta_0^2 a_{d-}^\dagger}{\sqrt{2}}, \\ b_d^{(p)} &\equiv \frac{B_d^* b_d - B_d b_{d-}^\dagger}{i|B_d|\sqrt{2}} = \frac{(R_2(\vartheta_0^*)^2 - R_1) b_d + (R_2 \vartheta_0^2 - R_1) b_{d-}^\dagger}{|R_2 \vartheta_0^2 - R_1|\sqrt{2}}. \end{aligned}$$

Substituting (A13, A14) into these formulas we finally obtain formulas (20, 21) for phase quadratures $a_d^{(p)}$, $b_d^{(p)}$.

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